

607 R / 607 E
Regular/Ex-Regular
M
(Arts / Science)
Section-A : (As per 2014 Syllabus)
Section-B : (As per 2011 to 2013 Syllabi)
**(Students are required to give their answers of
any one Section according to the Syllabus)**

SET – C

2014 (A)

ARTS / SCIENCE

MATHEMATICS

Full Marks - 100

Time : 3 Hours

SECTION – A

(As per 2014 Syllabus)

Answer all questions as per instructions given in each.

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନରେ ଦିଆଯାଇଥିବା ନିର୍ଦ୍ଦେଶ ଅନୁଯାୟୀ ସମ୍ପତ୍ତ ପ୍ରଶ୍ନ ଉଚିତ ଦିଆ ।

The figures in the right-hand margin indicate marks.

ବର୍ଣ୍ଣିତ ପାର୍ଶ୍ଵ ମାର୍କରେ ନମ୍ବର ପ୍ରତିଟି ହୋଲାଙ୍ଗି ।

P.T.O.

Electronic gadgets are not allowed in the examination hall.

ପରୀକ୍ଷା ହଲରେ ଇଲେକ୍ଟ୍ରୋନିକ୍ ସହିତ ବ୍ୟବହାର ନିଷେଧ ଥିଲା ।

Group - A

କ - ବିଭାଗ

(Marks - 10)

(୧୦ ନମ୍ବର)

1. Answer all the questions : $1 \times 10 = 10$
ସମ୍ପୂର୍ଣ୍ଣ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) Is 0 unique ?

୦ ଅନ୍ୟ ଅଟେ କି ?

(b) If a_{ij} is the element in the i th row and j th column of a 3rd order determinant whose value is 1 and C_{ij} is the cofactor of a_{ij} then what is the value of $a_{11}(C_{11} + C_{21}) + a_{12}(C_{12} + C_{22}) + a_{13}(C_{13} + C_{23})$?

ଯदି a_{ij} 1 ମୂଲ୍ୟ ବିଶିଷ୍ଟ ଏକ ତୃତୀୟ ବର୍ଗ ଡିଟରମ୍�ିନାନ୍ଟର i th
ଆତ୍ମି ଓ j th ତତ୍ତ୍ଵର ଉପାଦାନ ହୋଇଥାଏ ଓ C_{ij} a_{ij} ର ସହ-
ଉପାଦାନ, ତେବେ $a_{11}(C_{11} + C_{21}) + a_{12}(C_{12} + C_{22}) + a_{13}(C_{13} + C_{23})$ ର ମୂଲ୍ୟ କେତେ ?

(c) Under which conditions the straight line

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$$

intersects the plane $Ax + By + Cz + D = 0$ at a point other than (a, b, c) ?

କେଉଁ ସର୍ବରେ

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$$

ସରଳରେଖାଟି $Ax + By + Cz + D = 0$ ସମ୍ବନ୍ଧରେ
(a, b, c) ବ୍ୟତୀତ ଅନ୍ୟ ଏକ ବିନ୍ଦୁରେ ଛେଦ କରିବ ?

(d) Write a logarithmic function which is differentiable only in the open interval $(-1, 1)$.
କେବଳ $(-1, 1)$ ମୁକ୍ତ ଅଭାବରେ ଅବକଳନୀୟ ହୋଇଥିବା
ଏକ ଲଗାରିଥମିକ ଫଳନ ଲେଖ ।

(e) If an event A is independent of itself then what is $P(A)$?

A ନିଜଠାରୁ ସତ୍ୱ ଘଟଣା ହୋଇଥିଲେ $P(A)$ କେତେ ?

(f) What do you mean by integration ? Write your answer in one sentence.
ସମାକଳନର ଅର୍ଥ କ'ଣ ? ଗୋଟିଏ ବାକ୍ୟରେ ଉଚ୍ଚର ଲେଖ ।

(g) What is the value of

$${}^{20}C_3 + {}^{20}C_4 + {}^{20}C_5 + \dots + {}^{20}C_{17} ?$$

${}^{20}C_3 + {}^{20}C_4 + {}^{20}C_5 + \dots + {}^{20}C_{17}$ ର ମୂଲ୍ୟ କେତେ ?

- (h) Write the differential equation of the family of straight lines parallel to the y-axis.

y-অক্ষ সহিত সমান্তর হোলথুবা সরলরেখা কূলৰ অবকল
সমাকৰণ লেখ।

- (i) How many straight lines in space through the origin are equally inclined to the co-ordinate axes ?

ঘেঁশুৰে মূল বিন্দু দেৱ যাইথুবা ও স্থানাঙ্ক অক্ষমানক সহিত
সমান সমান আনত কেতোটি সরলরেখা হোলপাৰিব ?

- (j) Write a function which has both relative and absolute maximum at the point (1, 2).

(1, 2) বিন্দুত উভয় স্থানাঙ্ক ও পরম গৃহিষ্ঠ মান থুবা
এক ফাংকশন লেখ।

Group - B

শ - বিজ্ঞান

(Marks - 60)

(৩০ নম্বৰ)

2. Answer any five questions : $3 \times 5 = 15$

যেকোণৰি পাঞ্চটি প্ৰশ্নৰ উত্তৰ দিখ :

- (a) Evaluate :

মূল্য নিরূপণ কৰ :

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x}}.$$

- (b) Examine the differentiability of $\ln x^2$ for all real values of x.

x ৰ সমষ্টি বাস্তব সংখ্যা মূল্য পাই $\ln x^2$ ৰ অবকলনীয়তা
পৰাপৰা কৰ।

- (c) Interpret Lagrange's mean value theorem
geometrically.

লাগ্ৰাঞ্জ মাধ্যমান প্ৰমেয়ৰ জ্যামিতিক ব্যাখ্যা কৰ।

- (d) Show that no two normals to a parabola are parallel.

দৰ্শা আ যে এক পাৰাবোলা প্ৰতি যেকোণৰি দুইটি অৰ্ভিলম্ব
সমান্তর হোলপাৰিবে নহ'।

- (e) Does there exist a homogeneous function
 $u(x, y)$ of degree n such that $u(1, -1) = 1$,
 $u_{xx}(1, -1) = 2$, $u_{xy}(1, -1) = 1$ and
 $u_{yy}(1, -1) = -1$? Give reasons to your
answer.

ন ঘাত বিশিষ্ট এপৰি এক সকাতীয় ফাংকশন $u(x, y)$ আছি কি
যেপৰি কি $u(1, -1) = 1$, $u_{xx}(1, -1) = 2$,
 $u_{xy}(1, -1) = 1$ ও $u_{yy}(1, -1) = -1$ হোৰ ? কাৰণ
ওহু উত্তৰ লেখ।

(f) If $y = e^x \sin x$, then find y_n and $y_n(0)$.

$y = e^x \sin x$ ହେଲେ, y_n ଓ $y_n(0)$ ନିର୍ଣ୍ଣୟ କର ।

(g) If $y = \operatorname{cosec}^{-1} x$, then find $\frac{dy}{dx}$ and determine its value at $x = -2$.

$y = \operatorname{cosec}^{-1} x$ ହେଲେ $\frac{dy}{dx}$ ନିର୍ଣ୍ଣୟ କର ଓ $x = -2$ ରେ
ଏହାର ମୂଳ୍ୟ ନିରୂପଣ କର ।

3. Answer any five questions : $3 \times 5 = 15$
ଯେକୌଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉଭୟର ଦିଅ :

(a) Find the area of the region bounded by the curve $y = \sin^3 x$ and the straight lines $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$ and $y = 0$.

$y = \sin^3 x$ ବତ୍ର ଓ $x = -\frac{\pi}{4}$, $x = \frac{\pi}{4}$ ଏବଂ $y = 0$ ସରଳରେଖାମାନଙ୍କ ଦ୍ୱାରା ଆବଶ୍ୟକ କ୍ଷେତ୍ରଫଳ ନିର୍ଣ୍ଣୟ କର ।

(b) Find the differential equation whose general solution is $ax^2 + by = 1$, where a and b are arbitrary constants.

ଯେଉଁ ଅବକଳ ସମୀକରଣର ସାଧାରଣ ସମାଧାନ $ax^2 + by = 1$, ଯେଉଁଠାରେ a ଓ b ଯଦୃକ୍ଷା ଭାବରେ ନିଆଯାଇଥିବା ଧ୍ୱବକ ସଂଖ୍ୟା ଦାହା ନିର୍ଣ୍ଣୟ କର ।

(c) Integrate :

ସମାକଳନ କର :

$$\int \frac{dx}{x^2 + x^3}.$$

(d) Solve :

ସମାଧାନ କର :

$$(x+y) dy + (x-y) dx = 0$$

(e) Integrate :

ସମାକଳନ କର :

$$\int \frac{x e^x}{(1+x)^2} dx.$$

(f) Find the integrating factor of the following differential equation :

ନିମ୍ନ ପ୍ରଦର ଅବକଳ ସମୀକରଣର ସମାକଳନ ଗୁଣକ ନିର୍ଣ୍ଣୟ କର :

$$(1+y^2) dx + (x - e^{-\tan^{-1} y}) dy = 0.$$

(g) Evaluate :

ମୂଳ୍ୟ ନିରୂପଣ କର :

$$\int_0^2 [x^2] dx.$$

4. Answer any **five** questions : $3 \times 5 = 15$

যেকোণীয়ি পাঞ্চটি প্রশ্নের উত্তর দিঅ :

(a) Solve the following LPP :

নিম্ন প্রদত্ত LPP-র সমাধান কর :

$$\text{Maximize} \quad Z = 20x + 30y$$

(গরিষ্ঠ মান নির্ণয় কর)

$$\text{Subject to (যেপৰি কি)} \quad 3x + 5y \leq 15$$

$$x, y \geq 0.$$

(b) Find the equation of a plane parallel to the plane $2x - y + 3z + 1 = 0$ and at a distance of 3 units away from it.

$2x - y + 3z + 1 = 0$ এমতলারু 3 একক দূরতাৰে থুবা
ও এহা সহিত সমান্তৰ হোলথুবা এমতলৰ সমাকৰণ
নির্ণয় কৰ।

(c) If the equation

$$x^2 + y^2 + z^2 + 2\lambda x + 4(\lambda + 1)y + 6z + 10 = 0$$

represents a sphere then find the values of λ .

$$x^2 + y^2 + z^2 + 2\lambda x + 4(\lambda + 1)y + 6z + 10 = 0$$

এক গোলকৰ সমাকৰণ হোলথলে λ র মূল্যগুড়িক নির্ণয়
কৰ।

(d) If the sum of two unit vectors is a unit vector
then find the magnitude of their difference.

দুইটি একক বিশালৰ যোৱাপৰি এক একক বিশাল হৈলে
বিশাল দুইটিৰ বিয়োৱাপৰি ম্যানিহুয়ুভ নির্ণয় কৰ।

(e) ABCD is a parallelogram. Using vector
method prove that the line joining A and the
mid-point of BC intersects the diagonal BD
in the ratio 1 : 2.

ABCD এক সামৰণিক বিত্ত। বিশাল পদ্ধতি অনুসৰণ কৰি
প্ৰমাণ কৰ যে A বিত্ত ও BC-ৰ মধ্যবিহুলু যোগ কৰুথৰা
সৱলৱেজা BD কৰ্ষক 1 : 2 অনুপাতৰে ছেদ কৰে।

(f) Without using the equation of a straight line
find the co-ordinates of the foot of the
perpendicular drawn from the point (1, 1, 1)
on the line joining the points (1, 4, 6) and
(5, 4, 4).

সৱলৱেজাৰ সমাকৰণ ব্যবহাৰ ন কৰি (1, 1, 1)
বিত্ত (1, 4, 6) ও (5, 4, 4) বিহুবলু যোগ কৰুথৰা
সৱলৱেজা প্ৰতি লম্ব পাদবিহুল ঘানাক নিৰূপণ কৰ।

(g) Prove that the following vectors can never
be coplanar for any real value of λ :

প্ৰমাণ কৰ যে নিম্ন প্ৰদত্ত বিশালগুড়িক λ র যেকোণীয়ি
বাস্তৱ সংখ্যা মূল্য পাইঁ এক সমতকীয় হোলপাৰিবে
নাহি :

$$(\lambda + 1)\hat{i} + 2\hat{j} + \hat{k}, -\hat{i} + \lambda \hat{j} + \hat{k}, \lambda \hat{i} + \hat{j} + 3\hat{k}$$

- (h) Using the method of elimination find the symmetrical form of equation of the line $6x + 8y + 3z = 10$ and $x + 2y + z = 3$.

ଅପ୍ୟାରଣ ପଦ୍ଧତି ଅନୁସରଣ କରି $6x + 8y + 3z = 10$ ଓ
 $x + 2y + z = 3$ ସରଳରେଖାଦ୍ୱୟର ସମୀକରଣ ସମ୍ପର୍କରେ ନିର୍ଣ୍ଣୟ କର ।

5. Answer any **five** questions : $3 \times 5 = 15$

ଯେକୋଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଆ :

- (a) State the fundamental principle of counting and using it find out the solution of the following problem :

Five cities A, B, C, D, E are connected with each other by straight roads. What is the number of such roads ?

ଗଣନ ନିୟମର ମୌଳିକ ତଥ୍ୟ ଉଲ୍ଲେଖ କର ଓ ଏହାକୁ ବ୍ୟବହାର କରି ନିମ୍ନ ପ୍ରଶ୍ନର ସମାଧାନ କର ।

ପାଞ୍ଚଟି ନଗର A, B, C, D, E ପରସର ସହିତ ସିଧା ରାଷ୍ଟ୍ରାବ୍ଦୀ ସଂଯୋଗ କରାଯାଇଥାଏ । ଏପରି ରାଷ୍ଟ୍ରାମାନଙ୍କର ସଂଖ୍ୟା କେତେ ?

- (b) Find the greatest coefficient in the expansion of $(1+x)^{15}$.

$(1+x)^{15}$ ର ସଂପ୍ରସାରଣରେ ବୃଦ୍ଧତମ ସହଗତି ନିର୍ଣ୍ଣୟ କର ।

- (c) Show that for each real value of λ the system of equations

$$(\lambda + 3)x + \lambda y = 0$$

$$x + (2\lambda + 5)y = 0$$

has a unique solution.

ଦର୍ଶାଅ ଯେ λ ର ପ୍ରତ୍ୟେକ ବାନ୍ଧବ ସଂଖ୍ୟା ମୂଲ୍ୟ ପାଇଁ ସର୍ବ ସମୀକରଣ

$$(\lambda + 3)x + \lambda y = 0$$

$$x + (2\lambda + 5)y = 0$$

ଏକ ଅନନ୍ୟ ସମାଧାନ ଅଛି ।

- (d) If $A = \begin{bmatrix} 0 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 0 \end{bmatrix}$, then prove that

$$\det \{(I+A)(I-A)^{-1}\} = 1.$$

$$A = \begin{bmatrix} 0 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 0 \end{bmatrix}$$

ହେଲେ, ପ୍ରମାଣ କର ଯେ

$$\det \{(I+A)(I-A)^{-1}\} = 1$$

- (e) If A and B are square matrices of same order then show by means of an example that $AB \neq BA$ in general.

A ഓ B സ്ഥാന വർത്ത വർത്തമാന്ത്രിക്ക് ഹേഠെ ഉദാഹരണ ദ്വാരാ വർശാര ദിഥ യേ സാധാരണതഃ $AB \neq BA$ ।

- (f) A person draws three cards at random one after another from a pack of 52 cards. Find the probability that all these cards are spades.

ഒരു ലോക 52 പത താഴ മുതാരു ഗോട്ടാക പരേ ഗോട്ടി ചിനി പത താഴ ചാണിലേ । ഏമുള്ള ചിനി പത താഴ കലാപാനി ഹോളത്തുവാര സ്ഥാവാദ്യതാ നിർണ്ണയ കര ।

- (g) Find the number of ways in which 5 non-distinguishable balls can be put into 4 boxes placed in a row.

ഗോട്ടി ധാറ്റിരേ ഥുബാ 4 ചി ബാക്കരേ പരസ്വര മന്ത്രേ പാർക്ക നഥുബാ 5 ചി പേശു യേജേ ഉപായരേ രഷാധാരപാരിബ താഹാ നിർണ്ണയ കര ।

- (h) Two balls are drawn from a bag containing 6 red and 4 yellow balls. Find the probability that at least one of the balls is yellow.

6 ചി നാലി ഓ 4 ചി ഹലദിଆ രജാര പേശു ഥുബാ ഗോട്ടി മുണിരു ദുളച്ച പേശു നിആഗലാ । എഹി ദുളച്ച പേശു മന്ത്രു അടിക്കമരേ ഗോട്ടി ഹലദിଆ പേശു ഥുബാര സ്ഥാവാദ്യതാ നിർണ്ണയ കര ।

Group - C

സാഹിത്യ

(Marks - 30)

(30 നമ്പർ)

6. Answer any **one** question :

യേക്കോൺസി ഗോട്ടി പ്രശ്നര ഉരുര ദിഥ :

- (a) If (എ) $y = e^{m \sin^{-1}x}$ then (ഒബേ) prove that
(സ്ഥാപിക്കുന്ന യേ)

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$$

- (b) Find the minimum distance of a point on the

$$\text{curve } \frac{4}{x^2} + \frac{1}{y^2} = 1 \text{ from the origin.}$$

മൂല ക്രൂഡാരു $\frac{4}{x^2} + \frac{1}{y^2} = 1$ കു ഉപരേ ഥുബാ ദുരു മുദ്രണ
ദൂരതാ നിർണ്ണയ കര ।

7. Answer any **one** question :

യേക്കോൺസി ഗോട്ടി പ്രശ്നര ഉരുര ദിഥ :

- (a) Evaluate :

മൂല്യ നിരൂപണ കര :

$$\int_0^{\pi} \frac{x}{1+\sin x} dx.$$

(b) Solve :

ସମାଧାନ କର :

$$(x + \tan y) dy = \sin 2y dx$$

8. Answer any one question :

ସେକୋଣସି ଗୋଟିଏ ପ୍ରସ୍ତର ଭରର ଦିଅ :

(a) Prove by vector method that in any triangle ABC,

ଦିଶାକୁ ପ୍ରଶାଳାରେ ପ୍ରମାଣ କର ଯେ ସେକୋଣସି ତ୍ରିଭୁଜ ABC ରେ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

(b) Find the equation of the sphere inscribed in a tetrahedron whose faces are $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 1$.

ସେହି ଚତୁର୍ଭୁକ୍ତନର ପାର୍ଶ୍ଵଭୂତିକ $x = 0$, $y = 0$, $z = 0$ ଓ $2x + 2y + z = 1$ ସେହି ଚତୁର୍ଭୁକ୍ତନରେ ଅତିରିକ୍ଷେତ୍ର ଗୋଲକର ସମୀକରଣ ନିର୍ଣ୍ଣୟ କର ।

(c) Solve the following LPP :

ନିୟ ପ୍ରଦର �LPP ର ସମାଧାନ କର :

Maximize : $Z = 20x + 10y$

(ଗରିଷ୍ଠ ମାନ ନିର୍ଣ୍ଣୟ କର)

subject to : (ସେପରିକି) $x + 2y \leq 40$
 $3x + y \geq 30$
 $4x + 3y \geq 60$
 $x, y \geq 0$

7
—
2

9. Answer any one question :

ସେକୋଣସି ଗୋଟିଏ ପ୍ରସ୍ତର ଭରର ଦିଅ :

(a) Solve the following system of equations by the matrix inversion method :

ମାତ୍ରିକ ବିଲୋମୀ ପଢ଼ି ଦ୍ୱାରା ନିୟ ପ୍ରଦର ସହସମୀକରଣର ସମାଧାନ କର :

$$x + y + z = 4$$

$$2x - y + 3z = 1$$

$$3x + 2y - z = 1$$

(b) Prove that :

ପ୍ରମାଣ କର ଯେ :

$$C_1 - \frac{1}{2}C_2 + \frac{1}{3}C_3 - \dots + (-1)^{n-1} \frac{1}{n}C_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$$

(c) The probability of a shooter hitting a target is $\frac{4}{5}$. Find the minimum number of times he

must fire so that the probability of hitting the target at least once is greater than 0.999.

ଜଣେ ବନ୍ଦୁକଚାଳନାବାରୀଙ୍କର ଗୋଟିଏ ଲକ୍ଷ୍ୟରେ କରିବାର
ସମ୍ଭାବ୍ୟତା $\frac{4}{5}$ । ନିର୍ଣ୍ଣୟ କର ଯେ ତାକୁ ସରକିମ୍ବ କେତେଥର
ଶୁଳ୍କ ଚଳାଇବାକୁ ହେବ ଯେପରିକି ଅତିକମରେ ଥରେ ଲକ୍ଷ୍ୟରେ
କରିବାର ସମ୍ଭାବ୍ୟତା 0.999 ଠାରୁ ଅଧିକ ହେବ ।

SECTION – B

(As per 2011 to 2013 Syllabi)

Answer all questions as per instructions given in each.

ପ୍ରତ୍ୟେକ ପ୍ରଶ୍ନରେ ବିଆୟାଇଥିବା ନିର୍ଦ୍ଦେଶ ଅଛିଯାଏଇ ସମ୍ଭାବ୍ୟତା ପ୍ରଶ୍ନର ଉଚ୍ଚତା ଦିଅ ।

The figures in the right-hand margin indicate marks.

ତଥିଣ ପାର୍ଶ୍ଵ ମାର୍କିଙ୍ଗରେ ନମର ସୂଚିତ ହୋଇଥାଏ ।

Electronic gadgets are not allowed in the examination hall.

ପରୀକ୍ଷା ହଜାରେ ଇଲେକ୍ଟ୍ରୋନିକ ଯୋଗିର ବ୍ୟବହାର ନିଷେଧ ଆବଶ୍ୟକ ।

Group – A

କ – ବିଭାଗ

(Marks – 10)
(୧୦ ନମର)

1. Answer all the questions : $1 \times 10 = 10$

ସମ୍ଭାବ୍ୟତା ପ୍ରଶ୍ନର ଉଚ୍ଚତା ଦିଅ :

- (a) Write the value of a if $x^2 + y^2 - az^2 - 2x + 6y - 4z + 1 = 0$ is the equation of a sphere.

$x^2 + y^2 - az^2 - 2x + 6y - 4z + 1 = 0$ ଗୋଟିଏ ଗୋଲକର

ସମୀକରଣ ହେଲେ ଏଇ ମାନ ଲେଖ ।

- (b) If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \alpha \begin{vmatrix} b & c & a \\ a & b & c \\ c & a & b \end{vmatrix}$, write the value of α .

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \alpha \begin{vmatrix} b & c & a \\ a & b & c \\ c & a & b \end{vmatrix} \text{ হেলে } \alpha \text{-র মান } \\ \text{লেখি।}$$

- (c) In how many ways can four persons sit at a round table?

এক গোলাকার মেলকু পরিবেশন করি চারিজন ব্যক্তি কেনে প্রকারে বসিপারিবে?

- (d) Write the value of $\int e^x \sec x (1 + \tan x) dx$.

$$\int e^x \sec x (1 + \tan x) dx \text{ র মূল্য লেখি।}$$

- (e) Write the function which is its own derivative
নিজে নিজর অবকলক হোଉবা ফালনটি লেখি।

- (f) Write the unit vector in the direction of
 $\hat{i} - 2\hat{j} + 2\hat{k}$.

$\hat{i} - 2\hat{j} + 2\hat{k}$ র এমদিগৰে একক দিশাঙ্কটি লেখি।

- (g) Write the number of all possible outcomes on a simultaneous throw of three dice.

তিনিটো লুকু গোটিকু একসংগে গড়াললে সমুদায় কেতে গোটি সম্ভাব্য পরিণাম রহিছি উল্লেখ করি।

- (h) Write the degree of the differential equation

$$e^{\frac{dy}{dx}} = x^2.$$

অবকল সমাকরণ $e^{\frac{dy}{dx}} = x^2$ র ঘাত কেতে লেখি।

- (i) Write the equation of the plane parallel to the zx -plane and passing through the point $(2, -4, 0)$.

zx -সমতল সহ সমাতর তথা $(2, -4, 0)$ বিন্দুগামী সমতলটির সমাকরণ লেখি।

- (j) If $f'(x)$ changes sign from negative to positive as x crosses c , then which of the following is true?

(i) $f(c)$ is a local maximum.

(ii) $f(c)$ is a local minimum.

c কু x অভিক্রম করিবা সহিত $f'(x)$ যদি রশামূকবু ধনামূকরে পরিণত হুঁ এ, তেবে নিম্নোক্ত কেৱল উচ্চিতি সত্য?

(i) $f(c)$ এক খানায় বৃহত্তম মান।

(ii) $f(c)$ এক খানায় ক্ষুত্রতম মান।

Group – B

ഖ – ബിഭാഗ

(Marks – 60)

(ഓ നമ്മര)

2. Answer any **five** questions : $3 \times 5 = 15$

യേക്കോൺസി പാഞ്ചത്തി പ്രശ്നര ഉറര ദിഥ :

- (a) Prove that the line through the points $(3, 4, -5)$ and $(2, -3, 1)$ is parallel to the plane $4x + 2y + 3z + 9 = 0$.

പ്രമാണ കര യേ $(3, 4, -5)$ ഓ $(2, -3, 1)$ ബിഡുഗാമാ രേഖാ $4x + 2y + 3z + 9 = 0$ ഘമചല പ്രതി ഘമാട്ടരു അനേ ।

- (b) Find the equation of the sphere with centre at $(2, -1, 4)$ and touching the plane $2x - y - 2z + 6 = 0$.

$(2, -1, 4)$ ബിഡുംരെ കേന്ദ്ര ആക $2x - y - 2z + 6 = 0$ ഘമചലകു സർ കരുതും ഗോലകര ഘമ1കരണ നിർവ്വയ കര ।

- (c) Find a unit vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{k}$.

$\hat{i} + \hat{j}$ ഓ $\hat{i} - \hat{k}$ ദിശാകുദ്ദമ പ്രതി ലൈ ഹേരുതും ഗോടിക് ഏകക ദിശാകു നിർവ്വയ കര ।

- (d) Using vector method show that the points $A(2, 6, 3)$, $B(1, 2, 7)$ and $C(3, 10, -1)$ are collinear.

ദിശാകു പരിഗിരേ പ്രമാണ കര യേ $A(2, 6, 3)$, $B(1, 2, 7)$ ഓ $C(3, 10, -1)$ ബിഡുഗുടിക ഏകരേജയും ।

- (e) Prove that the points $(0, 1, 2)$, $(2, 5, 8)$, $(5, 6, 6)$ and $(3, 2, 0)$ form a parallelogram.

പ്രമാണ കര യേ $(0, 1, 2)$, $(2, 5, 8)$, $(5, 6, 6)$ ഓ $(3, 2, 0)$ ബിഡുഗുടിക ഏക സാമ്പത്രിക ചിത്ര ഗഠന കരാൻ ।

- (f) Find the maximum value of $Z = 20x + 30y$ and corresponding values of x and y subject to $3x + 5y \leq 15$ and $x, y \geq 0$.

$3x + 5y \leq 15$ ഓ $x, y \geq 0$ ഹേബേ, $Z = 20x + 30y$ റ റാഖി മാന ഏബു തെസ്റ്റുക x ഓ y റ മാന നിർവ്വയ കര ।

- (g) Find the equation of the plane passing through the intersection of the planes $2x + 3y - 4z + 1 = 0$ and $3x - y + z - 17 = 0$ and the point $(3, 2, 1)$.

$2x + 3y - 4z + 1 = 0$ ഓ $3x - y + z - 17 = 0$ ഘമചലഭ്യര ഛേദ തഥാ $(3, 2, 1)$ ബിഡുഗാമ1 ഘമചലരിത ഘമ1കരണ നിർവ്വയ കര ।

- (h) Find the vector \overrightarrow{PQ} , its magnitude and direction cosines if P and Q have co-

ordinates $(2, -1, -1)$ and $(-1, -3, 2)$ respectively.

P ഓ Q ദില്ലും ഘാനാക്ക് യഥാക്രമം $(2, -1, -1)$ ഓ
 $(-1, -3, 2)$ ഹേഠെ ദിഗാക്ക് \overline{PQ} , ഏഹാര ദേഖ്യം എബം
 ദിഗായ കൊച്ചി ജ്ഞാ ത്രസ്യ നിർണ്ണയ കര ।

3. Answer any five questions : $3 \times 5 = 15$

യേക്കോണ്ടി പാഞ്ച പ്രശ്നര ഉള്ള ദിഥ :

(a) Factorize :

ഉപാദകരെ പ്രകാശ കര :

$$\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix}$$

(b) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & x \end{bmatrix}$ and

$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, find the value of x.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & x \end{bmatrix} \text{ and } AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

ഹേഠെ, x റ മാന നിർണ്ണയ കര ।

(c) Prove that $2, 6, 10, \dots$ upto n^{th} factor =

$$\frac{(2n)!}{n!}$$
.

പ്രമാണ കര യേ $2, 6, 10, \dots n\text{-ഒന്ന് ഉപാദക പ്രധ്യാ$

$$= \frac{(2n)!}{n!}$$
 ।

(d) Prove that :

പ്രമാണ കര യേ :

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$

(e) A bag contains 4 black and 5 white balls from which 6 balls are drawn at random. Determine the number of ways in which at least 3 black balls can be drawn.

യഥി 4 ടി കലാ ഗോലി ഓ 5 ടി ധകാ ഗോലി ഥുബാ ഏക ബധാര മധ്യരു 6 ടി ഗോലി യടുക്കാ വാഹാര കരായാം, തേബേ സമുദായ കേതേ പ്രകാരരെ അതി കമ്പരേ 3 ടി കലാ ഗോലി വാഹാര കരായാളപാരിബ നിർണ്ണയ കര ।

- (f) A man has 6 friends. In how many ways can he invite two or more of them to a dinner party?

ଜଣେ ବ୍ୟକ୍ତିଙ୍କର 6 ଜଣ ବହୁ ଅଛନ୍ତି । ସେମାନଙ୍କ ମଧ୍ୟରୁ ଦୁଇ ବା ତା'ଠାରୁ ଅଧିକ ବହୁକୁ ସେ ସମ୍ବନ୍ଧାୟ କେତେ ପ୍ରକାରରେ ଏକ ନୈଶଭୋକ୍ତିକୁ ନିମନ୍ତଣ କରିପାରିବେ ?

- (g) Find the probability that all the 6 faces show different numbers when 6 dice are rolled.

୬ଟି ଲୁହୁ ଗୋଟିକୁ ଗଡ଼ାଇଲେ ମିଳୁଥିବା ସମସ୍ତ ୬ଟି ପାର୍ଶ୍ଵରେ ଭିନ୍ନ ଭିନ୍ନ ସଂଖ୍ୟା ଦେଖା ଦେବାର ସମ୍ଭାବ୍ୟତା ନିର୍ଣ୍ଣୟ କର ।

- (h) 4 girls and 4 boys sit in a row. Find the probability that the 4 girls are together.

4 ଜଣ ବାଳିକା ଓ 4 ଜଣ ବାଲକ ଗୋଟିଏ ଧାଡ଼ିରେ ବସନ୍ତ ସମସ୍ତ 4 ଜଣ ବାଳିକା ଏକତ୍ର ବସିବାର ସମ୍ଭାବ୍ୟତା କେଣେ ନିର୍ଣ୍ଣୟ କର ।

4. Answer any five questions:

$3 \times 5 = 15$

ଯେକୌଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

- (a) Find the extreme value of $y=xe^{-x}$ and state whether it is maximum or minimum.

$y=xe^{-x}$ ର ରତ୍ନ ମାନ ନିର୍ବ୍ୟାପଣ କରି ତାହା ଅଧ୍ୟକ୍ଷତମ କିମ୍ବା ନ୍ୟୂନତମ ଦର୍ଶାଅ ।

- (b) Differentiate $(\tan x)^x$ with respect to x .

$(\tan x)^x$ ର x ଭିତିକ ଅବଳକ ନିର୍ଣ୍ଣୟ କର ।

- (c) Evaluate :

ମାନ ନିର୍ଣ୍ଣୟ କର :

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x \cos x}$$

- (d) Find $\frac{dy}{dx}$ if $\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$.

$\sin x = \frac{2t}{1+t^2}$, $\tan y = \frac{2t}{1-t^2}$ ହେଲେ $\frac{dy}{dx}$ ନିର୍ଣ୍ଣୟ କର ।

- (e) If $z = \tan^{-1} \frac{x^3+y^3}{x+y}$, prove that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \sin 2z.$$

$z = \tan^{-1} \frac{x^3+y^3}{x+y}$ ହେଲେ, ପ୍ରମାଣ କର ଯେ

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \sin 2z.$$

- (f) Find the slope of the normal to the curve given by $x=3\cos\theta-\cos^3\theta$, $y=3\sin\theta-\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

$\theta = \frac{\pi}{4}$ ପାଇଁ ନିମ୍ନୋକ୍ତ ବକ୍ରର ସମ୍ପୂର୍ଣ୍ଣ ବିହୁରେ ଅଭିଲମ୍ବନ ଯୋଗ୍ୟ ନିର୍ଣ୍ଣୟ କର :

$$x=3\cos\theta-\cos^3\theta, y=3\sin\theta-\sin^3\theta$$

- (g) If $2y=x\left(1+\frac{dy}{dx}\right)$ prove that y_2 is a constant.
 $2y=x\left(1+\frac{dy}{dx}\right)$ ହେଲେ, ପ୍ରମାଣ କର ଯେ y_2 ଏକ ଧ୍ୱବକ ଅଛେ ।

5. Answer any five questions : $3 \times 5 = 15$

ସେକୌଣସି ପାଞ୍ଚଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

- (a) Evaluate :

ମାନ ନିର୍ଣ୍ଣୟ କର :

$$\int_{0}^{\frac{\pi}{2}} \frac{dx}{1 + \tan x}$$

- (b) Integrate :

ସମାକଳନ କର :

$$\int \frac{\ln x}{x^5} dx.$$

- (c) Integrate :

ସମାକଳନ କର :

$$\int \frac{\sec \theta \tan \theta}{\sec^2 \theta + 4} d\theta.$$

- (d) Solve :

ସମାଧାନ କର :

$$(1+x^2) \frac{dy}{dx} + 2xy = \cos x.$$

- (e) Form the differential equation whose general solution is $y = at + be^t$.

ସେହି ଅବଳକ ସମୀକରଣଟି ନିର୍ଣ୍ଣୟ କର ଯାହାର ବ୍ୟାପକ ସମାଧାନ $y = at + be^t$ ଅଛେ ।

- (f) Integrate :

ସମାକଳନ କର :

$$\int \frac{e^{4x}}{e^{8x} + 4} dx.$$

- (g) Solve :

ସମାଧାନ କର :

$$\frac{dy}{dx} = \frac{1}{x^2 - 7x + 12}.$$

Group – C
ગ – બિભાગ
(Marks – 30)
(૩૦ નમ્નર)

6. Answer any **one** question : 7 $\frac{1}{2}$

યેકોણથી ગોચિએ પ્રશ્નાનું ઉત્તર દિયું :

- (a) Find the equation of the sphere passing through the point $(3, -5, 9)$ whose centre is the point of intersection of the line

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$$

with the plane $x + y + z - 8 = 0$.

યેરું ગોલકટિ $(3, -5, 9)$ બિન્દુગામની ઓ યાહાર કેન્દ્રબિન્દુટિ

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{4}$$

રેખા ઓ $x + y + z - 8 = 0$ સમાન રેખાની કેન્દ્રબિન્દુ તાહાર સમાન નિર્ણય કરાયાની

- (b) Solve the following LPP graphically :

લેખચિત્ર યાહાયાયે નિર્મલિષ્ઠ છે LPP ર ઘણાધાન કરાયાની

Minimize : $Z = 20x_1 + 10x_2$

(ઘર્બનીય માન નિર્ણય કરાયાની)

subject to : (યેપરિકિ)

$$x_1 + 2x_2 \leq 40$$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1, x_2 \geq 0$$

- (c) If θ be the measure of the angle between the vectors $\hat{i} - 3\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$, find the value of $\sin \theta$.

$\hat{i} - 3\hat{j} + \hat{k}$ ઓ $\hat{i} + \hat{j} + \hat{k}$ દ્વારા બિન્દુએ મધ્ય કોણ પરિમાણ θ હેલે, $\sin \theta$ ર માન નિર્ણય કરાયાની

7. Answer any **one** question : 7 $\frac{1}{2}$

યેકોણથી ગોચિએ પ્રશ્નાનું ઉત્તર દિયું :

- (a) If A and B are two events such that $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, then find $P(A/B^C)$ and $P(B/A^C)$.

દ્વારા ઘણા A ઓ B પાછે $P(A) = 0.6$, $P(B) = 0.4$

ઓ $P(A \cap B) = 0.2$ હેલે, $P(A / B^C)$ ઓ

$P(B / A^C)$ નિર્ણય કરાયાની

- (b) Prove that :

પ્રમાણ કરું યે :

$$C_0 C_1 + C_1 C_2 + C_2 C_3 + \dots + C_{n-1} C_n = \frac{(2n)!}{(n-1)! (n+1)!}$$

(c) Solve by Cramer's rule :

କ୍ରାମରଙ୍କ ନିୟମ ସାହୀଯରେ ସମାଧାନ କର :

$$2x + y + 2z = 2$$

$$3x + 2y + z = 2$$

$$-x + y + 3z = 6$$

8. Answer any one question :

ଯେକୋଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) If $y = e^{m \cos^{-1} x}$ then prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$$

$y = e^{m \cos^{-1} x}$ ହେଲେ, ପ୍ରମାଣ କର ଯେ

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0.$$

(b) Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is a constant.

ପ୍ରମାଣ କର ଯେ $\sqrt{x} + \sqrt{y} = \sqrt{a}$ ବକ୍ର ପ୍ରତି ଯେକୋଣସି ସର୍ବକର ଯ୍ୟାନାଙ୍କ-ଅକ୍ଷ ଉପରିୟ ଛେଦାଂଶୁଭ୍ୟତର ସମାନ ଏକ ଧୂବକ ଅଛେ ।

Answer any one question :

7 $\frac{1}{2}$

ଯେକୋଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) Solve :

ସମାଧାନ କର :

$$(x^2 + y^2) dx - 2xy dy = 0$$

(b) Prove that :

ପ୍ରମାଣ କର ଯେ :

$$\int_0^1 \frac{\ln x}{\sqrt{1-x^2}} dx = \frac{\pi}{2} \ln \frac{1}{2}.$$